MT. DIABLO UNIFIED SCHOOL DISTRICT
COURSE OF STUDY

COURSE TITLE:	**Pre-Calculus**
COURSE NUMBER:	**1390**
CBEDS NUMBER:	**2414**
DEPARTMENT:	**Mathematics**
LENGTH OF COURSE:	**One Year**
CREDITS PER SEMESTER:	**5**
GRADE LEVEL(S):	**10 - 12**
REQUIRED OR ELECTIVE:	**This course fulfills one year of the high school mathematics requirement and UC/CSU "c" requirement.**
PREREQUISITES:	**Required - C or better Algebra II or Algebra II/Trig and/or Teacher Recommendation**

BOARD OF EDUCATION ADOPTION: ~~**June 22, 2010**~~

~~**COURSE DESCRIPTION:**~~

~~This course is designed to develop students' critical thinking in the analysis of functions and function modeling. Students will review and expand advanced algebra topics and apply problem-solving techniques using graphical, numerical, and analytical methods. Students will be able to accurately model various scenarios using an appropriate model and will develop their skills in choosing and justifying a model, as well as verifying results, using mathematical properties and real-world context. Students will leave this course with the skills and abilities to connect mathematics to the world around them and be successful in higher level mathematics courses.~~

~~**COURSE PURPOSE:**~~

~~Students will be able to apply the tools for mathematical modeling to linear, quadratic, polynomial and rational functions.~~
~~Students will be able to identify, construct, and analyze the relationship among various trigonometric functions and their graphs.~~
~~Students will be able to identify, construct, and analyze the relationship of conics, exponential functions, and logarithmic functions.~~
~~Students will be able to identify, analyze and construct models of finite and discontinuous quantities such as functions, logic, sequences, algorithms, matrices, and inductions.~~

COURSE OVERVIEW

Pre-Calculus focuses on the study of families of functions, their application in mathematical modeling, and the use of equivalence to rewrite expressions to reveal important features. Students analyze features of a variety of functions and their graphs, connect different representations, and identify and apply transformations of equations and graphs. To solve problems using function models, students choose among function families, fit linear and nonlinear functions to data, and interpret, apply, and evaluate the resulting models. The study of functions in this course includes strengthening of concepts and skills from prior courses, fuller development of equivalent forms of functions, and an in-depth study of trigonometry and its applications.

In addition to the California Common Core State Standards for Mathematics, students will experience and gain fluency with the 8 Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Overall, the quality of a learning environment depends on the extent to which it provides opportunities for students along the following five dimensions:

1. The richness of disciplinary concepts and practices (“the content”) available for learning;
2. Student sense-making and “productive struggle”;
3. Meaningful and equitable access to concepts and practices for all students;
4. Means for constructing positive disciplinary identities through presenting, discussion and refining ideas; and
5. The responsiveness of the environment to student thinking.

~~Unit 1 Relations, Functions and Graphs~~

~~Goals:~~

~~Student will be able to:~~
~~- Analyze graphs of functions~~
~~- Solve equations and systems of equations using multiple methods~~
~~-Identify and analyze families of functions~~
~~- Solve problems using equation models~~
~~- Find exact and approximate zeros and roots of functions and equations~~

~~Topics:~~

~~-Determine whether a given relation is a function and perform operations with functions -~~
~~Evaluate and find zeros of linear functions using functional notation~~
~~-Graph and write functions and inequalities~~

-Write equations of parallel and perpendicular lines
-Model data using scatter plots and write predictions equations
-Define add, subtract and multiply matrices
-Graph functions, relations, inverses, and inequalities
-Analyze families of graphs
-Investigate symmetry, continuity, end behavior, and transformations of graphs -Find asymptotes and extrema of functions
-Solve problems involving direct, inverse, and joint variation
-Determine roots of polynomial equations
-Solve quadratic, rational, and radical equations and rational and radical inequalities -Find the factors of polynomials
-Approximate real zeros of polynomial functions
-Write and interpret polynomial functions that model real-world data

Students will apply prior knowledge of linear and quadratic functions to the analysis of those functions and graphs, as well as extending these concepts to more complex polynomial and rational functions in order to identify zeros, roots, maxima, minima, points of inflection, end behavior, and symmetry. Students will analyze graphs and functions, identifying their parent function, as well as apply transformations.

Students will take real-world data and create matrices using matrix operations to assess changes in their data and analyze the results, focusing on metacognitive practices and innovative thinking. Unit 1 focuses on developing student perseverance in problem-solving and developing student expertise in mathematical modeling. An additional focus of this beginning unit is to encourage wonderment and awe of advanced mathematics.

Unit 2 Trigonometry

Goals:

Students will be able to:
Find missing angles, side lengths, and area of triangles
Graph trigonometric functions
Derive a unit circle based on special triangles
Verify trigonometric identities and solve trigonometric equations

Topics:

Convert decimal degree measures to degrees, minutes, and seconds and vice versa -Solve triangles
Find the values of trigonometric functions
Find the area of triangles
Change from radian measure to degree measure, and vice versa
Find linear and angular velocity
Use and draw graphs of trigonometric functions and their inverses
Find the amplitude, the period, the phase shift, and the vertical shift for trigonometric functions
Write trigonometric equations to model a given situation
Use reciprocal, quotient, Pythagorean, symmetry, and opposite-angle identities -

Verify trigonometric identities
Use sum, difference, double-angle, and half-angle identities
Solve trigonometric equations and inequalities

This unit begins with review of right angle trigonometry. Students will acquire knowledge of the basic sine, cosine, tangent ratios and functions and apply this to the solving of right triangles. Students will investigate given situations to determine appropriate use of trigonometric tools such as basic and inverse trigonometric functions and the *Law of Sines* and *Law of Cosines*.

Students derive the unit circle using radians and special triangles. The acquisition of these skills will enable students to define and graph the three foundational trigonometric functions sine, cosine, and tangent as well as the reciprocal functions cosecant, secant, and cotangent. Students will transform these functions, demonstrating how a change in parameter affects the period, amplitude and phase of the function.

Students will derive identities including the Pythagorean identities, sum and difference, double and half angles. Students will develop and enhance their logical thinking skills by verifying complex identities through simple, basic identities as well as solving trigonometric equations. By the end of the unit, students will have developed a conceptual understanding of the basic trigonometric functions, their inverses and reciprocals, and be able to use them in real world modeling. Unit 2 focuses on developing student expertise in mathematical modeling, using appropriate tools strategically, and attending to precision.

Unit 3 Advanced Functions and Graphing

Goals:

Students will be able to

Identify and graph conic sections using standard and general forms of equations · Simplify, evaluate, and graph exponential and logarithmic expressions, equations, and functions

Topics:

·Use analytic methods to prove geometric relationships

·Use and determine the standard and general forms of the equation for conic sections · Find equations, analyze and sketch the graphs of circles, parabolas , ellipses and hyperbolas.

Simplifying and evaluating expressions containing rational and irrational exponents. · Use and graph exponential functions and inequalities

Evaluate expressions and graph and solve equations involving logarithms · Model real-world situations and solve problems using common and natural logarithms.

Unit 3 starts with an overview of plane geometry. Students will decontextualize and find parametric equations from real-world applications. They will analyze parametric equations to determine the conic section they correspond to and graph the equations. They will be able to form parametric equations given characteristics of the graph.

Students will refine and broaden their comprehension of exponential and logarithmic functions from algebra II starting with reviewing simplification and evaluation of rational and irrational exponents. They will look to real-world situations and model them using tables and graphs in order to make predictions. They will analyze and decontextualize expressions, equations and

graphs from the broad range of real-world scenarios.

Unit 3 focuses on developing student's expertise in: making sense of problems and preserving in solving them, modeling with mathematics, attending to precision, and using appropriate tools strategically.

Unit 4 Discrete Mathematics

Goals:

Students will be able to:

-Analyze limits, sums, convergence, and divergence of sequences and series -

Solve problems involving combinations and permutations

-Find probabilities of real-world scenarios

-Display, interpret and analyze data

Topics:

-Find specified term(s) of arithmetic, geometric and infinite sequences. - Find sums of arithmetic, geometric, and infinite series.

Determine whether a series is convergent or divergent.

Use sigma notations

Use the Binomial Theorem to expand binomials

Solving problems involving combinations and permutations

Distinguish between independent and dependent events and between mutually exclusive and mutually inclusive events.

Find probabilities

Find odds for the success and failure of an event.

Make and use bar graphs, histograms, frequency distribution tables, stem-and-leaf plots and box-and-whisker plots.

Find the measures of central tendency and the measures of variability

Use the normal distribution curve.

Unit 4 begins with the review of arithmetic and geometric sequences. Students have to derive a formula written in summation notation for a geometric or arithmetic sequences from a repeated pattern and determine whether it's limit is divergent or convergent. They use binomial theorem to expand binomials and to find a specific term in it's expansion.

Students will enhance their skills in these sections with the experience of using realia and data such as dice, deck of cards, and statistical recordings to represent combinations and probability. Students will analyze real-world data and scenarios to determine what appropriate methods will be applied to combination and permutation problems. Students will increase their ability to differentiate between independent events, mutually exclusive events and complements.

Students will understand how graphs represent data and how to interpret the graphs. Student will be able to analyze the data and choose the appropriate graph or model. Students will learn about Normal distributions and central measures of tendency and be able to use this model to approximate a binomial distribution. Students will analyze data and scenarios to determine what

appropriate methods will be applied in both solving and graphing. They will increase their skills in measures of central tendency and apply them to the normal distribution curve. Unit 4 focuses on developing student's expertise in: modeling with mathematics, making sense of problems and persevere in solving them, looking for and expressing regularity in repeated reasoning and using appropriate tools strategically.

KEY ASSIGNMENTS:

Students will be assigned homework exercises from the text that supports the daily lessons. Reading, written analysis and in-depth problems will be assigned appropriately throughout the course to support abstract and quantitative reasoning. Students will apply their knowledge of mathematical analysis to various real-world scenarios

Unit 1: Relations, Functions and Graphs
Transformation Exploration: Students will explore functions and their corresponding graphs. They will be given various functions within a family of functions and compare the graphs of these functions using calculators and/or computer software. Based on these functions, students will make conjectures relating these functions and the transformations of the graphs. Students will support their conjectures graphically, algebraically, and numerically. In collaborative groups, students will compare and contrast their discoveries with other students, critique the reasoning of others through a peer-review process, and reach a general conclusion about transformations.

Unit 2: Trigonometry
Unit Circle: Students will work together collaboratively to make a unit circle. Students will incorporate prior knowledge of right triangle trigonometry to find the sine, cosine and tangent from the coordinates on the unit circle. Students will use this analysis to graph the sine, cosine and tangent functions. Students will make connections of these functions with real-world applications and present to the class, providing their analysis and mathematical reasoning with their function models.

Unit 3 Advanced Functions and Graphing
Modeling Medicine: Students will start this unit by studying the growth of cancerous tumor cells. Students will use scientific data to write models for tumor growth. They will use graphs and algorithms in data to study what happens when various treatments are used on tumors and use the results to evaluate the effectiveness of the treatments. Students will determine how functions can be used to determine which tumors are more dangerous by analyzing math models. At the end of the unit students will be assessed through their abilities to analyze a combination of graphs, functions, and data sets for a tumor's growth from diagnosis to after treatment to evaluate how destructive the tumor is and if the treatment is having any effect. Students will present their findings with the class and provide their mathematical reasoning and justification for their conclusions.

Unit 4 Discrete Mathematics
Data Survey: Students will work alone or in groups to describe variability in data. Students will create a survey and collect an original data set of values with precision at the interval or ratio level of measurements which provides the following information: small values, printed computer result of descriptive statistics and graphs, a written description of the purpose of the data, the

~~method used to gather the data and with important characteristics, pattern recognitions, deviations from patterns, distinguishing of quantitative and categorical data, and use of appropriate models. In the reflection and critiques of their conclusions, students will share their results to the class in an oral presentation and group discussion. Students will show mastery through their final analysis and summarizing their data. Students will become familiar with the complete data analysis process from data collection to interpretation of data, as well as improve their ability to think and work interdependently.~~

COURSE CONTENT:

Unit 1: Algebraic Structure, Polynomials, and Rational Expressions

A unit focused on "Algebraic Restructuring" reviews skills such as completing the square and factoring, and applies them to a variety of purposes such as the Rational Root Theorem, analyzing conic sections, and advanced solving techniques for equations involving exponential, logarithmic, and trigonometric functions.

Students will be able:

- Define and divide polynomials
- Determine the maximum number of zeros of a polynomial
- Find all rational zeros of a polynomial function
- Factor a polynomial completely
- Recognize and describe the graphs of various polynomial functions
- Identify the properties of general polynomial functions

At the end of the unit, students may develop a hypothetical business plan. They will complete a real planning template and will write a reflection of their work for each section. With this project, students will learn how math can be applied to financial planning in a real world application.

Unit 2: Sequences and Series

This unit addresses sequences in general, but places particular emphasis on arithmetic and geometric sequences which are expressed using sequence notation and vocabulary but examined in close connection with what students already know about linear and exponential functions. Series are also explored, including notation, vocabulary, the concept of convergence/divergence for infinite series, computation methods, and applications.

Students will be able:

- Interpret notation for sums/series
- Find the sum for an arithmetic series

At the end of the unit, students may complete the end of the unit assessment that includes interpreting notation for sums/series and connecting approaches to visualization of the arithmetic series sum formula.

Unit 3: Solutions of Equations, Inequalities, and Systems

The general solving concepts and skills emphasized in this unit include the meaning of solutions (equations, inequalities, systems), the relationship between analytic and graphical solutions, solving by undoing, and rewriting using equivalence (to assist in solving analytically). Specific solving techniques and considerations are emphasized during units focused on (1) exponentials and logs and (2) trigonometry. There are various possibilities in terms of learning progressions.

Students will be able:

- Solve simple trig equations
- Solve more complex trig equations involving algebraic equivalence such as factoring and

trigonometric equivalence

At the end of the unit, students may use a self assessment learning goal to solve a variety of equations which includes solving simple exponential and trigonometric equations, solve systems of 1 linear and 1 quadratic equations, and solve polynomial equations by applying the rational root theorem.

Unit 4 Functions and Function Families
The functions topics emphasized in this course are multiple representations (tables, graphs, equations, verbal), features of functions, function composition and decomposition, inverses, and transformations of functions. The major families studied are polynomial (including linear and quadratic), exponential, logarithmic, sinusoidal, and tangent functions, although other families are also included (e.g. linear absolute value, radical, rational, other trig functions). Due to the central role of multiple representations, function features, and transformations for this course overall, all versions of the course revisit these topics in multiple units throughout the year, in continuation of the focus on families of functions.

Students will be able:

- Apply transformations to families of functions (graph↔equations)
- Apply and compare multiple representations of functions, including equations in different forms
- Given the features of a function, sketch a graph

At the end of the unit, students will continue to work on their business plan and create corresponding graphs of their financial plan in order to pitch their business to the class. Students will generate a final reflection of their process, including areas for revision.

Unit 5: Modeling
The modeling skills identifies for this course are:

- Define quantities of interest when modeling
- Describe the features of function families and choose among function families for modeling purposes (FOCUS: exponential, quadratic, simple radical, simple rational, sinusoidal, piecewise-defined)
- Apply and compare multiple representations of functions in modeling contexts
- Apply right triangle trigonometry, Law of Sines, and Law of Cosines to solve triangles
- Evaluate models

Students will be able to:

- Define quantities of interest when modeling
- Choose among function families for modeling purposes
- Apply and compare multiple representations of functions in modeling contexts
- Evaluate models

At the end of the unit, students may work on a modeling task from Desmos.org. Students will be asked to generate questions based on the image of a charging cell phone. Students will be asked to make a scatterplot, choose a function family, write a model equation, and then predict how charged the phone will be at a given time.

Unit 6: Statistics and Probability
In this unit the required topics are finding equations for nonlinear trendlines, and interpretation of linear and nonlinear trendlines. Optional topics include trendline related topics such as residuals

and correlation analysis, probability topics, and inference-related topics such as simulations, statistical significance in experiments, and margin of error. The required skills can be incorporated in different ways.

Students will be able to find equations for nonlinear trendlines. Students will be able to:

- Choose among function families for modeling purposes
- Apply multiple representations of functions in modeling contexts
- Evaluate models

At the end of the unit, students may complete the end of the unit assessment that includes students deriving equations for nonlinear trendlines, choose among function families for modeling purposes, apply multiple representations of functions in modeling contexts, and evaluate models.

~~**INSTRUCTIONS METHODS and/or STRATEGIES:**~~

~~Teachers will use multiple modalities and instructional methods to meet the learning needs of our diverse student population with a focus on integrating the eight mathematical practices and the sixteen habits of mind for 21st century learning.~~

~~Reading assignments, direct instruction and class discussions are used to introduce students to new concepts and terminology, show example problems and explain methods and reasoning. Reading assignments often are abstract, so the students are required to take that information and put it into context for the class. This leads to class discussion where they will make arguments and critique the arguments of others. The reading assignments also help them look for and make use of structure and patterns. In addition, class discussions may be inquiry-based, which will promote critical thinking skills, challenge students' thinking, and increase their listening skills with understanding and perception of another's point of view.~~

~~Example problems in class will help motivate the students with real-world problems and connect the students to their interests. It also will help them see the patterns in problems when they work on the homework which will reinforce their skills and provide more unique real-world problems. These problems are also of a higher level requiring perseverance and precision. Homework is used to reinforce the concepts and knowledge in the daily lessons. Homework will also assist students in attending to precision, developing pattern recognition, and practicing making use of structure.~~

~~The small group work and technology applications provide the students the need to determine the appropriate tools, be able to model the complex real situation with a simpler model, and increase their ability to think interdependently as teams.~~

~~During introduction to new concepts emphasis is placed on perseverance, making sense of problems, precision, striving for accuracy, and making use of structure.~~

~~Explorations will allow students to examine conjectures, test and analyze hypotheses, reflect on results, test structures, and deconstruct patterns. Explorations are also focused on encouraging creativity, imagination, and innovation in students.~~

~~Once students have the foundation, then small group work, technology and projects are used to provide hands-on experience and increase the students' ability to apply their knowledge and improve their critical thinking skills. Emphasis is placed on reasoning abstractly and~~

~~quantitatively, constructing viable arguments and critiquing the reasoning of others, using appropriate tools and mathematical modeling, questioning, and problem posing.~~

~~Problem Solving scenarios will be assigned and routinely analyze and interpret data to explore and deepen their understanding of mathematical concepts. An additional focus will be on creating scenarios that increase students' intrigue and interest in the world around them.~~

~~Real-life modeling will be used to describe a situation and interpret the results in the context of the problem and will encourage students to remain life-long learners.~~

COURSE MATERIALS

Authors	Edition	Publisher	Title	Website
Gordon-Holliday/Holliday et al.	2006	Glencoe	Advanced Mathematical Concepts: Pre-Calculus with Applications	

Teacher support resources can also be found in the Educational Services Website and supplemental online curriculum (for ex. Apex).

~~ASSESSMENTS INCLUDING METHODS and/or TOOLS~~

~~The assessments are in the form of quizzes, tests, and projects. There are basic quizzes and portion of tests that are in multiple-choice format, which target basic knowledge of terms and calculations. Multiple-choice questions will include multiple correct or application concepts that foster higher thinking skills. The tests also include short constructed response portions which require the students to formulate a hypothesis and draw conclusions as well as use the appropriate skills to solve the problem.~~

~~Projects require the students to design and create their own study and apply concepts learned throughout the year. They will also be required to communicate the findings of their project with the class, as well as critique the projects of other students.~~

~~Some of the formative assessments make sure that the students can identify different structures and recognize key concepts as well as collect and display data. Some of the bigger quizzes require that they compare concepts or differentiate between situations, as well as apply these concepts to different situations.~~

~~The tests, group scenarios, and projects require the students to analyze a given situation and apply the concepts to prove the hypothesis true. They may also need to create a situation where they would need to use a specific inferential tool. The projects provide time for the students to work collaboratively, present an argument and critique the logic of others. The projects are designed to connect the concepts learned throughout the chapter or book, as well as their metacognition skills and flexibility in thinking.~~

EVALUATION OF STUDENT PROGRESS:

Assessment Methods:

- Summative assessment
- Formative Assessment

Formative:

- Mathematical Discourse
- Reflection questions
- Teacher observations/evidence
- Student discussions
- Quiz
- Exit ticket

Summative:

- Performance task
- Unit Assessment

~~Committee Members:~~

~~1. Dan Leingang Teacher College Park H~~
~~2. Michelle HIgby Teacher Concord HS~~
~~3. Wendy Rounds Teacher Northgate HS~~
~~4. Luis Miguel Soto Teacher Olympic HS~~
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