MT. DIABLO UNIFIED SCHOOL DISTRICT
COURSE OF STUDY

COURSE TITLE:	**Algebra II/ Trigonometry**
COURSE NUMBER:	**1340**
CALPADS NUMBER:	**2408**
CST:	**Algebra II**
DEPARTMENT:	**High School Mathematics**
~~**NCLB CREDENTIAL**~~	
REQUIREMENT:	**Math Credential with Subject Matter Proficiency**
LENGTH OF COURSE:	**One Year**
CREDITS PER SEMESTER:	**5**
GRADE LEVEL(S):	**10-12**
REQUIRED OR ELECTIVE:	**This course fulfills one year of the high school mathematics requirement and UC/CSU "c" requirement.**
PREREQUISITES:	**Successful completion of Geometry with a C or better.**

BOARD OF EDUCATION ADOPTION: ~~**June 22, 2010**~~

~~**COURSE DESCRIPTION:**~~

~~This course provides the student with the knowledge, concepts, skills identified in the California State Math Standards for Algebra II and Trigonometry. The ability to communicate mathematical reasoning and understanding will be incorporated into all topics. In addition, students will develop their ability to construct formal, logical arguments in algebraic and trigonometric settings and problems. This course is the third course in the three-year mathematics requirement for four-year college admission.~~

COURSE OVERVIEW

In this course, students expand understanding of expressions including rewriting, interpreting and examining rational, radical, polynomial expressions and deriving the formula of the sums of finite geometric series. Students continue expanding their knowledge of rational, polynomial, radical, exponential and logarithmic functions. They learn to represent functions algebraically, graphically, in numerical tables and by verbal descriptions. Students will extend their knowledge of functions by examining domains, various discontinuities, and through the use of advanced theorems such as the binomial expansion theorem, Descartes rules of signs, and the conjugate root theorem. Students will also be exposed to more sophisticated

and complex applications.

Special emphasis in this course is given to the expansion of trigonometric topics including degree to radian conversion, to interpreting radian measure of angles in the unit circle, graphing all six trigonometric functions, modeling the periodic phenomena of trig functions, and proving/evaluating trig identities. Students in this course will spend additional time working with trigonometric applications and inverse trigonometric functions, as well as solving trigonometric equations which require proficiency with identities. Students will also be using modeling to represent real-world applications.

Students continue expanding their knowledge of statistics by summarizing, representing, and interpreting data using the normal distribution. Moreover, students make inferences and justify conclusions based on sampling, experiments and observational studies.

In addition to the California Common Core State Standards for Mathematics, students will experience and gain fluency with the 8 Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Overall, the quality of a learning environment depends on the extent to which it provides opportunities for students along the following five dimensions:

1. The richness of disciplinary concepts and practices (“the content”) available for learning;
2. Student sense-making and “productive struggle”;
3. Meaningful and equitable access to concepts and practices for all students;
4. Means for constructing positive disciplinary identities through presenting, discussion and refining ideas; and
5. The responsiveness of the environment to student thinking.

~~**COURSE OUTLINE:**~~

~~**1. MAJOR GOALS**~~

~~1.1 To develop the ability to reason logically and think symbolically~~
~~1.2 To develop skills for communicating mathematically~~
~~1.3 To build algebraic and trigonometric models, formulate and solve problems~~
~~1.4 To Improve the skills necessary to be successful in multiple careers~~

~~**2. PERFORMANCE OBJECTIVES:**~~

~~(numbers in parentheses refers to an appropriate California State Standard)~~
~~2.1 Algebra II~~
~~2.1.1 Solve equations and inequalities involving absolute value. (Algebra II, 1.0)~~
~~2.1.2 Solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices. (Algebra II, 2.0)~~

2.1.3 Operations on polynomials, including long division. (Algebra II, 3.0)
2.1.4 Factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes. (Algebra II, 4.0)
2.1.5 Demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the
plane. (Algebra II, 5.0)
2.1.6 Add, subtract, multiply, and divide complex numbers. (Algebra II, 6.0)
2.1.7 Add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator. (Algebra II, 7.0)
2.1.8 Solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Application of these techniques in solving word problems. Solving quadratic equations in the complex number system. (Algebra II, 8.0)
2.1.9 Demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is determination of how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x-b)^2$. (Algebra II, 9.0)
2.1.10 Graphing quadratic functions and determines the maxima, minima, and zeros of the
function. (Algebra II, 10.0)
2.1.11 Prove simple laws of logarithms. (Algebra II, 11.0) 2.1.12 Understanding of the inverse relationship between exponents and logarithms and use this relationship to solve
problems involving logarithms and exponents. (Algebra II, 11.1)
2.1.13 Judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step. (Algebra II, 11.2)
2.1.14 Knowledge of the laws of fractional exponents, exponential functions and the use
of these functions in problems involving exponential growth and decay. (Algebra II, 12.0)
2.1.15 Use the definition of logarithms to translate between logarithms in any base. (Algebra II, 13.0)
2.1.16 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. (Algebra II, 14.0)
2.1.17 Determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true. (Algebra II, 15.0)
2.1.18 Demonstration and explanation of how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it. (Algebra II, 16.0)
2.1.19 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, demonstration of knowledge of the method of completing the
square in order to put the equation into standard form and recognition of whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Graphing of the equation. (Algebra II, 17.0)
2.1.20 Demonstration of fundamental counting principles to compute combinations and permutations. (Algebra II,18.0)
2.1.21 Demonstration of computing probabilities by using combinations and permutations. (Algebra II,19.0)
2.1.22 Knowledge of the binomial theorem and usage of it to expand binomial expressions that are raised to positive integer powers. (Algebra II,20.0)
2.1.23 Application of the method of mathematical induction to probe general statements

~~about hc positive integers. (Algebra II, 21.0) 2.1.24 Finding the general term and the sums of arithmetic series and of both finite and infinite geometric series. (Algebra II, 22.0)~~

~~2.1.25 Derivation of the summation formulas for arithmetic series and for both finite and infinite geometric series. (Algebra II, 23.0)~~

~~2.1.26 Solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions. (Algebra II, 24.0)~~

~~2.1.27 Use of properties from number systems to justify steps in combining and simplifying functions. (Algebra II, 25.0)~~

~~2.2 Trigonometry~~

~~2.2.1 Understand the notion of angle and how to measure it, in both degrees and radians. Conversion between degrees and radians. (Trigonometry, 1.0)~~

~~2.2.2 Knowledge of the definition of sine and cosine as *y*- and *x* coordinates of points on~~

~~the unit circle and are familiar with the graphs of the sine and cosine functions. (Trigonometry, 2.0)~~

~~2.2.3 Knowledge of the identity $\cos^2(x) + \sin^2(x) = 1$. (Trigonometry, 3.0)~~

~~2.2.4 Prove that this identity, $\cos^2(x) + \sin^2(x) = 1$, is equivalent to the Pythagorean Theorem (i.e. Prove this identity by using the~~

~~Pythagorean Theorem and, conversely, prove the Pythagorean~~

~~Theorem as a consequence of this identity). (Trigonometry, 3.1) 2.2.5 Prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, using this identity to prove that $\sec^2(x) = \tan^2(x) + 1$. (Trigonometry, 3.2) 2.2.6 Graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift. (Trigonometry, 4.0)~~

~~2.2.7 Knowledge of the definitions of the tangent and cotangent functions and can graph them. (Trigonometry, 5.0)~~

~~2.2.8 Knowledge of the definitions of the secant and cosecant functions and can graph them. (Trigonometry, 6.0)~~

~~2.2.9 Knowledge of tangents and that the tangent of an angle that a line makes with the *x*-axis is equal to the slope of the line. (Trigonometry, 7.0)~~

~~2.2.10 Knowledge of the definitions of the inverse trigonometric functions and graphing of the functions. (Trigonometry, 8.0)~~

~~2.2.11 Computation, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points. Trigonometry, 9.0)~~

~~2.2.12 Demonstration of an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities. (Trigonometry, 10.0)~~

~~2.2.13 Demonstration of an understanding of half angle and double angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric~~

~~identities . (Trigonometry, 11.0) 2.2.14 Use trigonometry to determine unknown sides or angles in right triangles. (Trigonometry, 12.0)~~

~~2.2.15 Knowledge of the laws of sines and the law of cosines and application of those laws to solve problems. (Trigonometry, 13.0)~~

~~2.2.16 Determination of the area of a triangle, given one angle and the two adjacent sides. (Trigonometry, 14.0)~~

~~2.2.17 Ability to use trigonometry in a variety of applications and word problems.(Trigonometry, 19.0)~~

~~3. CONTENT OUTLINE:~~

~~(numbers in parentheses refer to appropriate performance objectives)~~

~~3.1 Algebra II~~
~~3.1.1 Equations or inequalities and systems of equations or inequalities including career applications (2.1.1, 2.1.2)~~
~~3.1.2 Polynomials, complex numbers, and rational expressions (2.1.3 2.1.8, 2.1.17)~~
~~3.1.3~~
~~Quadratic formula, completing the square and quadratic functions (2.1.8 2.1.10, 2.1.18, 2.1.19)~~
~~3.1.4 Logarithms with career applications (2.1.11 2.1.13, 2.1.15 2.1.16, 2.1.17)~~
~~3.1.5 Fractional exponents (2.1.14)~~
~~3.1.6 Conic Sections (2.1.16)~~
~~3.1.7 Combinations and permutations (2.1.20 2.1.21)~~
~~3.1.8 Binomial theorem (2.1.22)~~
~~3.1.9 Mathematical induction (2.1.23)~~
~~3.1.10 Algebraic and geometric series (2.1.24 2.1.25)~~
~~3.1.11 Functions (2.1.26 2.1.27)~~
~~3.2 Trigonometry~~
~~3.2.1 Angles in degrees and radians (2.2.1)~~
~~3.2.2 Sine, cosine, tangent, cotangent, secant and cosecant functions (2.2.2, 2.2.6, 2.2.7 2.2.9, 2.2.12 2.2.13)~~
~~3.2.3 Trigonometric identities (2.2.3 2.2.5)~~
~~3.2.4 Inverse trigonometric functions (2.2.10 2.2.11)~~
~~3.2.5 Trigonometry in triangles (2.2.14, 2.1.16)~~
~~3.2.6 Law of cosines and Law of sines (2.2.15)~~
~~3.2.7 Mathematical induction (2.1.23)~~
~~3.2.8 Applications and word problems with **career** emphasis (2.2.21)~~

COURSE CONTENT:

Unit 1: Linear Functions
Transformation of linear, quadratic, and absolute value functions are explored. The parent functions are established and then transformed functions are compared to the parent. Rigid transformations include vertical and horizontal translations and reflections. Non-rigid transformations are vertical and horizontal stretches and shrinks. Students will review modeling with linear functions which involves writing linear functions from given information and fitting a line to data. Students will solve linear systems, building upon skills from algebra 1. Students will apply knowledge of systems to solve linear optimization problems.

At the end of the unit, students may be given some situations or data sets to plot and/or solve. Students will demonstrate mastery of relationships within one linear equation, as well as interactions between multiple equations in a system. Applications to cost or efficiency can be demonstrated.

Unit 2: Quadratic Functions
There are four common forms in which quadratics are written, and each gives information about the graph and the behavior of the function. Understanding the connection between the characteristics of a quadratic and it's equation can help students apply their knowledge to when working with real life applications. Students will explore the vertex of quadratic functions as a means to quickly distinguish the type of transformations displayed in a graph. Additionally, students will look at characteristics of

quadratic functions and their graphs including lines of symmetry, maximum and minimum points, and how the quadratic is defined by the fixed point (focus) and a fixed line (directrix). Students will examine modeling using quadratic functions and use a graphing calculator to perform regression analysis.

At the end of the unit, students may complete an economics-based project that investigates demand functions for a concert. They will analyze constraints, such as decreasing demand with increasing ticket price, and will find the optimum price that will maximize revenue for the venue. Students will be able to model the function using technology, such as a graphing calculator, and verify their results.

Unit 3: Quadratic Equations and Complex Numbers
Students will use five strategies for solving quadratic functions: graphing, square rooting, factoring, completing the square, and the Quadratic Formula. This unit will require students to make informed decisions as to which strategy is the most efficient. Students will define complex numbers and their operations so that they may move into solving quadratics with imaginary solutions. Students will solve nonlinear systems and graph systems of nonlinear inequalities. Students will compare solving nonlinear systems graphically and analytically.

At the end of the unit, students will complete a unit test, which includes using the different strategies for solving quadratic functions and solving quadratics with imaginary solutions.

Unit 4: Polynomial Functions
Polynomial functions are studied in detail here, starting with polynomial operations. Pascal's triangle and the binomial expansion theorem are explored. Prior knowledge of factoring is extended to include 3 and 4th degree polynomials. Synthetic division and the application of the Remainder Theorem is used to find rational roots. Graphs of polynomials and their transformations will be explored using technology. Students will use graphs and equations to determine discontinuities, find roots using division, use graphs and equations to model real-life data.

At the end of the unit, students will complete a unit test, which includes creation and analysis of polynomial functions and graphs, optimizations given constraints, finding the real roots of a function, and finding asymptotes for rational functions.

Unit 5: Rational Exponents and Radical Functions
This unit introduces radicals and nth roots, and rational exponents. These concepts are connected to the rules of exponents from Algebra 1 noting the exponents are no longer restricted to non-zero whole numbers. The graphs of radical functions are used as a tool to help students think about solutions of radical equations and inequalities. Students will be able to state the domain of a rational function and differentiate between point and infinite discontinuity. Solving radical equations is revealed as an extension of solving other functions with the addition of the concept of extraneous solutions. Function operations and inverses of radicals will be explored as well. Students will graph and transform radical functions and their inverses.

At the end of the unit, students will prove the Pythagorean theorem using radicals for legs and the hypotenuse. This project can include modeling as well as computations involving rational and irrational numbers, and any combination. Students can factor expressions in many ways, and compare with the quadratic formula, to solve for complex roots. Students demonstrate mastery of radicals. The Pythagorean theory, and

complex roots.

Unit 6: Exponential and Logarithmic Functions
This unit presents exponential and logarithmic functions. Students will write and graph logarithmic functions using the common base 10, e, and other bases. These types of functions will be applied to real-life applications such as: growth, decay, and interest type of problems. Students will progress to solving exponential and logarithmic equations using different approaches: analytical, and graphical. Students will work with a real data set and model it using logarithmic or exponential functions.

At the end of the unit, students may integrate exponential functions with chemistry when investigating the concept of half-life of a radioactive substance. They will create a decaying exponential function that can represent the remaining quantity of a radioactive substance. Students can investigate real world data for radioactive substances and use their function to find how much time needs to pass in order for the amount of mass of a radioactive substance to reach levels safe for humans.

Unit 7: Rational Functions
Rational functions are explored in this unit. The unit uses the simplest rational function, inverse variation, to introduce the concept. Graphs of rational functions will be used to identify horizontal and vertical asymptotes. Students will extend this experience to identifying these asymptotes from the actual rational expression without looking at the graphs. Operations with rational functions will be linked to operations with fractions. Although the operations are primarily analytical they will be confirmed using a graphing calculator.

At the end of the unit, students will complete a unit test, which includes identifying horizontal and vertical asymptotes with the use of graphs and operations with fractions.

Unit 8: Sequences and Series
This unit will extend the students' knowledge and use of arithmetic and geometric sequences. The students will explore sums and partial sums of infinite geometric series both numerically and graphically. The students will also explore recursive sequences as well with connections to linear and exponential functions as well.

At the end of the unit, Students may work collaboratively to analyze an electrical engineering task and ultimately write a rule using sequences of knowledge obtained in the unit to describe the situation.

Unit 9: Trigonometric Ratios and Functions
This unit takes our study of functions to the trigonometric functions and as such begins with a review of the trigonometric ratios and right triangle geometry. Students will use radians and the six trigonometric functions defined by the unit circle. Students will be able to determine the 6 trig ratios for angles which are multiples of 30° and 45° (or in radians) as well as triangles defined by a point (x,y) on the terminal side. Graphs of sine, cosine and tangent are developed by plotting functional values for benchmark angles and the concept of periodic functions is introduced. The graphs of the remaining four trigonometric functions are deduced from knowing the relationships between these functions and sine and cosine. Knowledge of transformations is used to plot functions, including amplitude, period change, vertical shift and phase shift. Students will understand the relationship between sine and cosine functions as a phase shift. Students will create and analyze sinusoidal models of real-life data. Students will graph and use the inverse trigonometric functions, but understanding domain restrictions.

Inverse functions will be used to solve trigonometric equations. The students will also be introduced to the trigonometric identities, including quotient identities, even and odd identities, Pythagorean identities, half and double angle formulas and the sum and difference formulas. Students will evaluate and simplify trigonometric expressions toward the use of solving complex trigonometric equations which require knowledge of identities. Students will also use identities to verify and prove other identities.

At the end of the unit, students may apply angle of elevation and othe trigonometry concepts to measure the height of nearby buildings. Students will use appropriate tools and technology to gather measurements of nearby buildings. This can be repeated for a few buildings, and reported in a professional report.

Unit 10: Probability
This unit starts with the classical concept of sample space and probability. As students develop their skills they will calculate the probability of independent and dependent events and use two way tables to calculate marginal frequencies. Permutations and combinations will be used to calculate the probabilities of compound events and to solve real-life problems. The students will explore the binomial and normal distributions as well, including use of the binomial expansion theorem to calculate binomial probability.

At the end of the unit, students may complete a probability and statistics project where they design and conduct a survey, and interpret the data using multiple methods. They can compare measures of central tendency and justify their conclusions as to the best measure. They will need to design their questions or survey in a way to minimize errors, analyze, and then graphically demonstrate their findings.

Unit 11: Data Analysis and Statistics
Data collection and analysis will be explored in this unit. The normal distributions will be used to calculate z scores and find the associated probabilities of events from the normal table. The concepts of experimental design to avoid bias will be covered. Students will understand the difference between an observational study and experiment and that correlation does not determine causality.

At the end of the unit, students will use data to make inferences. Students will become more analytic as they read or hear accounts of research or claims.

~~4.~~ TIME ESTIMATES:
~~4.1~~ Instructional sequences vary in length from a few days to several weeks.

~~5. INSTRUCTIONAL MATERIALS:~~
~~5.1 District adopted textbooks~~
~~5.2 Supplementary and teacher-created materials that include a career focus 5.3 Technology materials~~

COURSE MATERIALS

Authors	Copyright	Publisher	Title	Website
Ron Larson & Laurie Boswell	2015	Big Ideas Learning, LLC	Algebra and Trigonometry	Bigideasmath.com

Teacher support resources can also be found in the Educational Services Website and supplemental online curriculum (for ex. Apex).

~~6.~~ EVALUATION OF STUDENT PROGRESS~~:~~

~~Students communicate mathematically and demonstrate content knowledge in a variety of ways that lead to mathematical competence in their chosen careers. 6.1 Teacher observation~~

~~6.2 Written assignments and projects~~

~~6.3 Quizzes and tests~~

~~6.4 Rubrics~~

Assessment Methods:

- Summative assessment
- Formative Assessment

Formative:

- Mathematical Discourse
- Reflection questions
- Teacher observations/evidence
- Student discussions
- Quiz
- Exit ticket

Summative:

- Performance task
- Unit Assessment

Committee Members:

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