MT. DIABLO UNIFIED SCHOOL DISTRICT
COURSE OF STUDY
DRAFT

COURSE TITLE:	**Advanced Math Topics**
COURSE NUMBER:	**9831**
CALPADS NUMBER:	**2417**
CST:	**Summative Math**
DEPARTMENT:	**Mathematics**
NCLB TEACHER CREDENTIAL REQUIREMENT:	**Math Credential with Subject Matter Proficiency**
LENGTH OF COURSE:	**One Year**
CREDITS PER SEMESTER:	**5**
GRADE LEVEL(S):	**10-12**
REQUIRED OR ELECTIVE:	**This course fulfills one year of the high school mathematics requirement and UC/CSU "c" requirement.**
PREREQUISITES:	**Successful completion of Algebra II or Algebra II/Trig**

BOARD OF EDUCATION ADOPTION:

COURSE DESCRIPTION:

This course is a study of advanced algebra topics, trigonometry, and discrete mathematics. The content includes relations, functions, polynomials, conic sections, logarithms, trigonometric functions and identities, sequences and series, matrices, and probability and statistics. This course is designed for students who have completed Algebra II or for students who have struggled in Algebra II/Trig in preparation for Pre-Calculus, Statistics or College Level Mathematics.

COURSE OUTLINE:

1. MAJOR GOALS

1.1 To review techniques and skills previously learned from the study of Algebra and Geometry.
1.2 To expand mathematical content and concepts in advance Algebra.
1.3 To strengthen their conceptual understanding of problems and mathematical reasoning and problem-solving.
1.4 To prepare the students for college level mathematics
1.5 To gain an awareness of the relevance of advance mathematics to a variety of careers.

2. PERFORMANCE OBJECTIVES:

2.1 ALGEBRA II
(The numbers in parentheses refer to the Algebra II California Mathematics Content Standards)

2.1.1 Students solve equations and inequalities involving absolute value. (1.0)

2.1.2 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices. (2.0)

2.1.3 Students are adept at operations on polynomials, including long division. (3.0)

2.1.4 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes. (4.0)

2.1.5 Students add, subtract, multiply, and divide complex numbers. (6.0)

2.1.6 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator. (7.0)

2.1.7 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system. (8.0)

2.1.8 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x-b)^2 + c$. (9.0)

2.1.9 Students graph quadratic functions and determine the maxima, minima, and zeros of the function.(10.0)

2.1.10 Students prove simple laws of logarithms.(11.0)

> 2.1.10.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (11.1)
>
> 2.1.10.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step. (11.2)

2.1.11 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay. (12.0)

2.1.12 Students use the definition of logarithms to translate between logarithms in any base. (13.0)

2.1.13 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. (14.0)

2.1.14 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true. (15.0)

2.1.15 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it. (16.0)

2.1.16 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation. (17.0)

2.1.17 Students use fundamental counting principles to compute combinations and permutations. (18.0)

2.1.18 Students use combinations and permutations to compute probabilities. (19.0)

2.1.19 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers. (20.0)

2.1.20 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series. (22.0)

2.1.21 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series. (23.0)

2.1.22 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions. (24.0)

2.2 LINEAR ALGEBRA
(The numbers in parentheses refer to the Linear Algebra California Mathematics Content Standards)

2.2.1 Students perform addition on matrices and vectors. (4.0)

2.2.1 Students perform matrix multiplication. (5.0)

2.2.3 Students demonstrate an understanding that linear systems are inconsistent (have no solutions), have exactly one solution, or have infinitely many solutions. (6.0)

2.2.4 Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane. (8.0)

2.2.5 Students demonstrate an understanding of the notion of the inverse to a square matrix and apply that concept to solve systems of linear equations. (9.0)

2.2.6 Students compute the determinants of 2 x 2 and 3 x 3 matrices and are familiar with their geometric interpretations as the area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in two-dimensional and three-dimensional spaces. (10.0)

2.2.7 Students know that a square matrix is invertible if, and only if, its determinant is nonzero. They can compute the inverse to 2 x 2 and 3 x 3 matrices using row reduction methods or Cramer's rule. (11.0)

2.3 MATHEMATICAL ANALYSIS

(The numbers in parentheses refer to the Mathematical Analysis California Mathematics Content Standards)

2.3.1 Students know the statement of, and can apply, the fundamental theorem of algebra. (4.0)

2.3.2 Students are familiar with conic sections, both analytically and geometrically: (5.0)

2.3.2.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth). (5.1)

2.3.2.2 Students can take a geometric description of a conic section - for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6 - and derive a quadratic equation representing it. (5.2)

2.4 TRIGONOMETRY

(The numbers in parentheses refer to the Trigonometry California Mathematics Content Standards)

2.4.1 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians. (1.0)

2.4.2 Students know the definition of sine and cosine as *y*-and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions. (2.0)

2.4.3 Students know the identity $\cos^2(x) + \sin^2(x) = 1$: (3.0)

2.4.3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity). (3.1)

2.4.3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$. (3.2)

2.4.4 Students graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift. (4.0)

2.4.5 Students know the definitions of the tangent and cotangent functions and can graph them. (5.0)

2.4.6 Students know the definitions of the secant and cosecant functions and can graph them. (6.0)

2.4.7 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line. (7.0)

2.4.8 Students know the definitions of the inverse trigonometric functions and can graph the functions. (8.0)

2.4.9 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points. (9.0)

2.4.10 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities. (10.0)

2.4.11 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities. (11.0)

2.4.12 Students use trigonometry to determine unknown sides or angles in right triangles. (12.0)

2.4.13 Students know the law of sines and the law of cosines and apply those laws to solve problems. (13.0)

2.4.14 Students determine the area of a triangle, given one angle and the two adjacent sides. (14.0)

2.4.15 Students are adept at using trigonometry in a variety of applications and word problems. (19.0)

2.5 PROBABILITY AND STATISTICS

(The numbers in parentheses refer to the Probability and Statistics California Mathematics Content Standards)

2.5.1 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces. (1.0)

2.5.2 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces. (2.0)

2.5.3 Students demonstrate an understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses. (3.0)

2.5.4 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families. (4.0)

2.5.5 Students determine the mean and the standard deviation of a normally distributed random variable. (5.0)

2.5.6 Students know the definitions of the mean, median, and mode of a distribution of data and can compute each in particular situations. (6.0)

2.5.7 Students compute the variance and the standard deviation of a distribution of data. (7.0)

2.5.8 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots. (8.0)

3. CONTENT OUTLINE:

3.1 Linear relations and functions
- 3.1.1 Identification of relations and functions
- 3.1.2 Evaluation and composition of functions
- 3.1.3 Graphs of linear equations and inequalities
- 3.1.4 The writing of linear equations
- 3.1.5 Applications of linear equations

3.2 Systems of linear equations and inequalities
- 3.2.1 Solutions to systems of equations in two and three variables
- 3.2.2 Matrix operations and applications
- 3.2.3 Graphs of systems of inequalities and applications

3.3 The nature of graphs
- 3.3.1 Families of graphs
- 3.3.2 Symmetries and transformations of graphs
- 3.3.3 Direct, inverse, and joint variation
- 3.3.4 Inverses, continuity, extrema, and end behavior of relations and functions
- 3.3.5 Graphs of rational functions

3.4 Polynomial and rational functions
- 3.4.1 Roots of polynomial equations
- 3.4.2 Solutions of quadratic equations
- 3.4.3 Factors of polynomials
- 3.4.4 Solutions of rational equations and inequalities
- 3.4.5 Solutions of radical equations and inequalities

3.4.6 The writing of polynomial functions

3.5 Conics
- 3.5.1 Standard and general forms of the equation of a circle
- 3.5.2 Standard and general forms of the equation of an ellipse
- 3.5.3 Standard and general forms of the equation of a hyperbola
- 3.5.4 Standard and general forms of the equation of parabola
- 3.5.5 Graphs of conic sections
- 3.5.6 Systems of second degree equations and inequalities

3.6 Exponential and logarithmic functions
- 3.6.1 Properties of exponents and logarithms
- 3.6.2 Simplification of expressions with rational exponents
- 3.6.3 Solutions of equations containing rational exponents and logarithms
- 3.6.4 Graphs of exponential and logarithmic functions and inequalities
- 3.6.5 Applications of exponents and logarithms

3.7 Trigonometric functions
- 3.7.1 Conversion and identification of degree measures
- 3.7.2 Values of trigonometric ratios
- 3.7.3 Values of six trigonometric functions
- 3.7.4 The use of trigonometry to solve right triangles
- 3.7.5 Inverses of trigonometric functions
- 3.7.6 The use of Law of Sines and Cosines to solve triangles
- 3.7.7 The use of Law of Cosines to find the area of a triangle

3.8 Graphs of trigonometric functions
- 3.8.1 Lengths of arcs and areas of sectors
- 3.8.2 Graphs of six trigonometric functions
- 3.8.3 The writing of sine and cosine equations
- 3.8.4 Transformations of sine and cosine functions
- 3.8.5 Applications of sine and cosine functions
- 3.8.6 Graphs of the inverses of sine, cosine, and tangent functions

3.9 Trigonometric identities and equations
- 3.9.1 Identification and use of trigonometric identities
- 3.9.2 Solutions of trigonometric equations and inequalities

3.10 Sequences and series
- 3.10.1 Arithmetic sequences and series
- 3.10.2 Geometric sequences and series
- 3.10.3 Convergence or divergence of series
- 3.10.4 The use of sigma notation
- 3.10.5 Expansion of binomials

3.11 Combinatorics and probabilities
- 3.11.1 Problems involving the Basic Counting Principle
- 3.11.2 Identification of independent, dependent, and mutually exclusive events
- 3.11.3 Problems involving permutations and combinations

3.11.4 Problems involving probability and odds
3.11.5 Definitions of basic probability principles

3.12 Statistics and data analysis
3.12.1 Development and interpretation of bar graphs and histograms
3.12.2 Identification and use of measures of central tendency
3.12.3 The use of stem-and-leaf plots and box-and-whisker plots for data analysis
3.12.4 Analysis of normal distributions
3.12.5 Calculation and evaluation of the standard error of the mean

4. TIME ESTIMATES:

4.1 Advanced Algebra topics – 50%

4.2 Trigonometry – 25%

4.3 Discrete Mathematics – 25%

5. INSTRUCTIONAL MATERIALS:

5.1 Board of Education Adopted Textbooks

5.2 Supplemental and teacher-created materials that may include a career focus

5.3 Technology materials

6. EVALUATION OF STUDENT PROGRESS:

6.1 Teacher observation

6.2 Written assignments and projects

6.3 Quizzes and tests

6.4 Rubrics

Committee Members:

Frank Bruketta	CVHS
Susan Seeley	CVHS
Daniel Leingang	CPHS
Khoa Phan	CPHS
Rianne Pfaltzgraff	NHS
Jasmin Mumford	YVHS
Sandy Bruketta	Curriculum Specialist (Curriculum & Instruction)